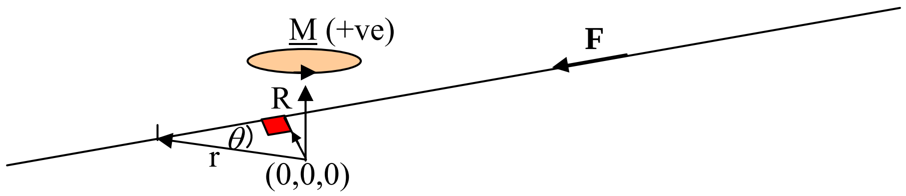
* NB. Vector Factoring described here is the basis of Vector Cryptography.
It is my personal invention copyright registered at Stationery Hall London in about 2000.

Vector Factorising.

The notion of factoring a vector came to me from seeing the **Force Line** at work.



M is the moment (turning effect) imposed on the vertical axis (passing through (0,0,0)) by the force F acting at any point 'r' on the line.

R is the effective crank(lever length) of 'r' and equals $|r|$ sine θ.

M ≡ $|F|\,|r|$ sine θ which is the same thing as the magnitude of the cross-product of F × r . M = (F × r) has the direction shown.

(r is the position of any point on the line, F is a 'sliding' vector that has the same effect at any point of application defined by r).

All of this is incidental to 'factoring' but the model suggests that the line of F could be ***any*** direction in the plane and r could be any point in the plane on the line. This model suggests that the pairs of F and r can be likened to pairs of factors i.e. a factor and a cofactor (similar to the factors of a composite nymber) that when used as the operands of a cross-product always give the same result i.e. M .

This suggests that if a vector is proposed as being the normal vector of a plane that passes through the origin than there are pairs of vectors within the plane (there for the finding) that when taken in a certain order are 'factors' per se of that vector. The experiment to find these factors is described here.

The current methodology of vector arithmetic is,

1) Vector addition.
2) Subtraction.
3) Multiplication in the Dot Product.
4) Multiplication in the Vector or Cross Product.
5) Multiplication by a scalar.
I am adding one more,
6) Factorising of a Vector.

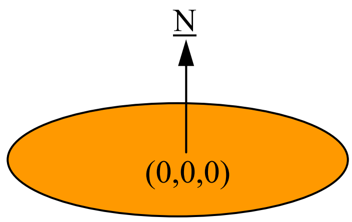
Vector division is not generally defined in vector methodology.

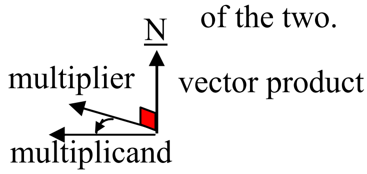
The vectors I am dealing with are three-dimensional ones used to represent physical quantities such as displacement - the 'types' of these quantities are not taken into account and only the number work of the factoring process is demonstrated.

Vector factoring.

A vector for factoring (factorising) is proposed as being the defining 'normal' vector of a plane - always shown here as a horizontal model but the reader should understand that it will be inclined in any attitude in space in practice.

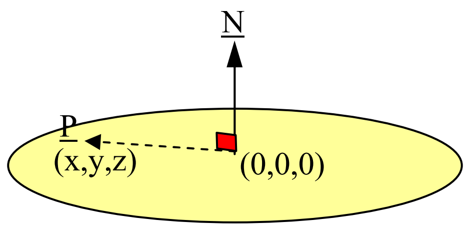
These are ordered 'factors' of N which is the vector or 'cross' product of the two.





The plane passes through the origin i.e. it contains the point (0,0,0).

Equation of the plane.



The point P (x,y,z) represents ***any*** point in the plane.
The direction ratios of the vector P relative to origin at (0,0,0) are (x-0), (y-0), (z-0).

Lemma_1.

The dot product of two mutually perpendicular vectors is always zero => P •N = 0.

So, Letting N = $\begin{pmatrix} \alpha \\ \beta \\ \gamma \end{pmatrix}$ and P = $\begin{pmatrix} (x-0) \\ (y-0) \\ (z-0) \end{pmatrix}$

$$\underline{P} \bullet \underline{N} = \begin{pmatrix} \alpha \cdot x \\ \beta \cdot y \\ \gamma \cdot z \end{pmatrix} - \begin{pmatrix} \alpha \cdot 0 \\ \beta \cdot 0 \\ \gamma \cdot 0 \end{pmatrix} \rightarrow (\alpha \cdot x + \beta \cdot y + \gamma \cdot z) = 0$$

This is the standard equation for a plane that passes through the origin, i.e. the equation of the plane is always.

$$(\alpha \cdot x + \beta \cdot y + \gamma \cdot z) = 0$$

i.e. this is the rule that must be satisfied by any point claming to be 'in' the plane.

Lemma_2.

Two non-parallel planes intersect along a straight line.

Harking back for a moment to two dimensions such as specifically the (X,Y) plane the axes may be intersected by a straight line. The intercept is a *point. The adage then is "Y intercepts occur when x = 0" and " X intercepts occur when y = 0". Importantly the intercept is a point.

In three dimensions however the axes per se are instead the bounding planes of the frame of reference, furthermore, the intercept is now a ***line*** that emanates from the intercept of each of the containing planes of the frame of reference as they meet with the inclined plane defined by $\underline{N}$.

Lemma_3
Two non-parallel planes intercept along a straight line.

The Factors.
The factors are named $\underline{V}_0$ (VeeZero) and $\underline{V}_1$ (VeeOne) - these are a seeding pair that lead to multiple factor lines.

Recapping on the discussion model to hand. I have three orthogonal planes that comprise the standard reference frame that is being intercepted by the inclined plane that is jointly defined by $\underline{N}$ (a defining normal vector) and the origin at (0.0.0).

Finding $\underline{V}_0$ (at the ZY intercept).

I need to find a point in the inclined plane,
the equation of the plane is,

$$(\alpha \cdot x + \beta \cdot y + \gamma \cdot z) = 0$$

I will guess that there is a point that has its x coefficient = 0 (certain to be the case? when $\underline{N}$ has three non-zero coefficients).

$$\text{So, } (\alpha \cdot 0 + \beta \cdot y + \gamma \cdot z) \quad = 0$$

$$(\beta \cdot y + \gamma \cdot z) = 0$$

For this to be true either,

1) y = $-\gamma$ and z = $+\beta$

or

2) y = $+\gamma$ and z = $-\beta$

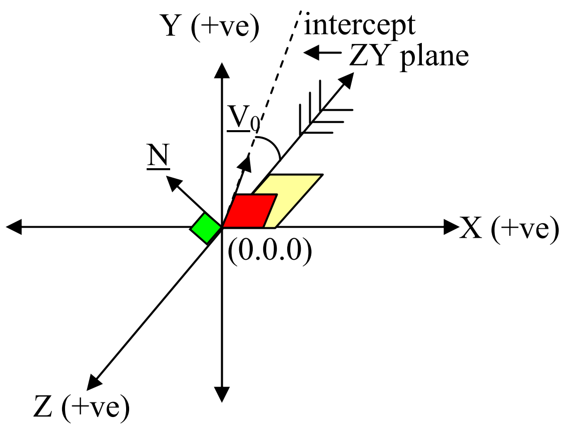
These are two options here that the reader may take. I have opted for 2) and this will always be used in these notes here in future. Note, option 1) is not to be discarded completely – it can be used additionally so as to provide an extra directed number-line if that is ever required in some special case in the future.

So, $\underline{V}_0$ can be written algebraically as $\begin{pmatrix} 0 \\ \gamma \\ -\beta \end{pmatrix}$ when x is put at 0.

I do not want $\underline{V}_0$ to have any scalar take-out factor so I divide throughout by the GCD of $\{\beta, \gamma\}$ and I shall call this GCD 'ε_x' (Epsilon_x <= x = 0).

$$\rightarrow \underline{V}_0 = \begin{pmatrix} 0 \\ \gamma/\varepsilon_x \\ -\beta/\varepsilon_x \end{pmatrix}$$

* This is the standard form that $\underline{V}_0$ will always take in the future.



To find the cofactor $\underline{V}_1$.

$\underline{V}_1$ needs to be found next. This is the position vector of another point nearby to $\underline{V}_0$, that is also in the plane defined by $\underline{N}$ such that $\underline{V}_1 \times \underline{V}_0 = \underline{N}$.

$$\text{Putting } V_1 = \begin{pmatrix} x \\ y \\ z \end{pmatrix} \text{ for the time being,}$$

$$\underline{V}_1 \text{ x } \underline{V}_0 = \underline{N} \rightarrow \begin{pmatrix} x \\ y \\ z \end{pmatrix} \times \begin{pmatrix} 0 \\ \gamma/\varepsilon_x \\ -\beta/\varepsilon x \end{pmatrix} = \begin{pmatrix} \alpha \\ \beta \\ \gamma \end{pmatrix}$$

$$\begin{vmatrix} i & j & k \\ x & y & z \\ 0 & \gamma/\varepsilon_x & -\beta/\varepsilon_x \end{vmatrix} = \begin{pmatrix} - \\ - \\ \gamma \end{pmatrix}$$

Solving for the k minor of the determinant gives me all I need to know here, Finding γ on the RHS $\rightarrow (x \times \gamma/\varepsilon_x) - (y \times 0) = \gamma \rightarrow x = \varepsilon_x$

$x = \varepsilon_x$ .... (fallout information from solving the 'k' minor of the determinant))

(I need to find y and z next)

Again, by the dot product,

$$\underline{V}_1 \bullet \underline{N} \rightarrow \begin{pmatrix} x \\ y \\ z \end{pmatrix} \cdot \begin{pmatrix} \alpha \\ \beta \\ \gamma \end{pmatrix} = \alpha \cdot x + \beta \cdot y + \gamma \cdot z = 0 \text{ (the equation of the plane once more)}$$

$$1)\ z = -(\alpha \cdot x + \beta \cdot y)/\gamma \quad \text{or } 2)\ y = -(\alpha \cdot x + \gamma \cdot z)/\beta$$

Again, in this instance I also have two options as previously with finding $\underline{V}_0$

Taking option 1) and collecting terms,

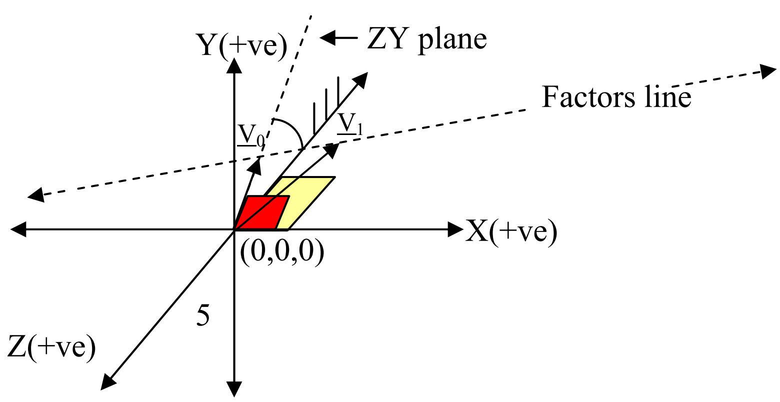
$$x = \varepsilon_x$$
$$y = y$$
$$z = -(\alpha \cdot x + \beta \cdot y)/\gamma$$

$$\underline{V}_1 = \begin{pmatrix} \varepsilon_x \\ y \\ -(\alpha \cdot \varepsilon_x + \beta \cdot y)/\gamma \end{pmatrix}$$

(y decides z here - the first integer for y modulo γ will do fine but any integer modulo γ is perfectly acceptable also.)

$$\text{Altogether then, } \underline{V}_0 = \begin{pmatrix} 0 \\ \gamma/\varepsilon_x \\ -\beta/\varepsilon_x \end{pmatrix} \quad \underline{V}_1 = \begin{pmatrix} \varepsilon_x \\ y \\ -(\alpha \cdot \varepsilon_x + \beta \cdot y)/\gamma \end{pmatrix}$$

This is the preferred form of these two 'primary' factors (not prime factors) that evolve from considering the ZY intercept only. I shall always use these in the future unless otherwise stated for all workings.



Similar working is also possible by taking the other two intercepts into consideration (-that working is not being repeated here but please see appendix – A, and B),

1) i.e. letting $\underline{V}_0 = \begin{pmatrix} x \\ y \\ z \end{pmatrix}$ and putting y = 0 so that $\underline{V}_0 = \begin{pmatrix} x \\ 0 \\ z \end{pmatrix}$

2) i.e. letting $\underline{V}_0 = \begin{pmatrix} x \\ y \\ z \end{pmatrix}$ and putting z = 0 so that $\underline{V}_0 = \begin{pmatrix} x \\ y \\ 0 \end{pmatrix}$

This gives two more remaining results.

It is necessary now to prove this result.

Proof that this pair V_1 x V_0 (Evolving from the ZY intercept) = N.

The proof is simply to multiply out $\underline{V}_1$ and $\underline{V}_0$ algebraically in the vector or cross product method of vector multiplication.

The order of multiplication is always $\underline{V}_1 \times \underline{V}_0$

$$\begin{pmatrix} \varepsilon_x \\ y \\ -(\alpha.\varepsilon_x + \beta.y)/\gamma \end{pmatrix} \times \begin{pmatrix} 0 \\ \gamma/\varepsilon_x \\ -\beta/\varepsilon_x \end{pmatrix} = \begin{pmatrix} \alpha \\ \beta \\ \gamma \end{pmatrix}$$

using the conventional determinant method in the computing of a cross-product.

$$\begin{vmatrix} i & j & k \\ \varepsilon_x & y & -(\alpha.\varepsilon_x + \beta.y)/\gamma \\ 0 & \gamma/\varepsilon_x & -\beta/\varepsilon_x \end{vmatrix}$$

$$(+)\ i\begin{vmatrix} y & -(\alpha.\varepsilon_x+\beta.y)/\gamma \\ \gamma/\varepsilon_x & -\beta/\varepsilon_x \end{vmatrix} = -\beta.y/\varepsilon_x + \alpha + \beta.y/\varepsilon_x = \alpha$$

$$(-)\ j\begin{vmatrix} \varepsilon_x & -(\alpha\cdot\varepsilon_x+\beta\cdot y)/\gamma \\ 0 & -\beta./\varepsilon_x \end{vmatrix} = -\beta$$ (the -ve j minor of the determinant becomes + with change-of-sign convention → $+\beta$.)

$$(+)\ k\begin{vmatrix} \varepsilon_x & y \\ 0 & \gamma/\varepsilon_x \end{vmatrix} = \gamma$$

This demonstrates that $\begin{pmatrix} 0 \\ \gamma/\varepsilon_x \\ -\beta/\varepsilon_x \end{pmatrix}$ and $\begin{pmatrix} \varepsilon_x \\ y \\ -(\alpha.\varepsilon_x+\beta.y)/\gamma \end{pmatrix}$ are indeed proper primary algebraic factors of $\underline{N}$.

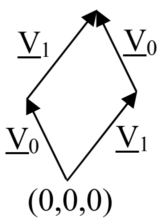
Lemma_4.

Equations of a straight line.

A particular line is uniquely located in space if,

a) it has a known direction and it passes a known point,

b) it passes through two known points.



Equations of lines that can be defined immediately from any pair V_0 and $\underline{V}_1$ as found initially,

Factor Line 1 :- $\underline{V}n = \underline{V}_0 + n\,(\underline{V}_1 - \underline{V}_0)$

Factor Line 2 :- $\underline{V}n = \underline{V}_0 + n\,(\underline{V}_1 + \underline{V}_0)$

Factor Line 3 :- $\underline{V}n = V_0 + n\,(V_1)$

Factor Line 4 :- $\underline{V}n = \underline{V}_1 + n\,(-\,\underline{V}_0)$

Factor Lines.

These factors of '$\underline{N}$' are being called 'primary' because they are a 'seeding' pair that is extensible to the full infinite family of factors that exists for any '$\underline{N}$'.
Family indeed is the word for it. The pairs of vectors $\underline{V}_0$ and $\underline{V}_1$ wherever they are found initially are each a single salient result on their own but their main use is to define general factor lines that are indeed usable also as directed number lines in cryptography on this occasion whereby the numbers on the lines can be defined by the corresponding vector factors of $\underline{N}$ as position vectors of numbers as displacements in space, not necessarily in pairs. Although discussed as pairs here in the realisation of factors of $\underline{N}$ each one of the pair is a valid *standalone* factor of $\underline{N}$ in its own right . In general, only integer values of numbers are used here in these notes although the methodology is valid for both float and integer values alike.

It becomes a matter of using various ploys (mainly while programming) that suggest themselves easily to the user to explore the plane looking for fresh directions and points that can be used to create more and more factor lines. I repeat, although these factors of $\underline{N}$ are being talked about here in pairs each one of the pair is a standalone factor of $\underline{N}$ in its own right and can be quoted as such.

Collected Equations of basic Factor Lines

Factor Line 1 :- $\underline{V}n = \underline{V}_0 + n\,(\underline{V}_1 - \underline{V}_0)$
Factor Line 2 :- $\underline{V}n = \underline{V}_0 + n\,(\underline{V}_1 + \underline{V}_0)$
Factor Line 3 :- $\underline{V}n = V_0 + n\,(V_1)$
Factor Line 4 :- $\underline{V}n = \underline{V}_1 + n\,(- \underline{V}_0)$

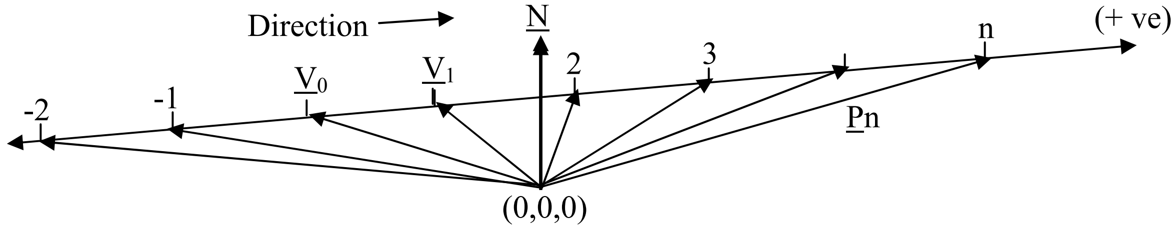
Recursion.

Any pair of factors $\underline{V}(n\text{-}1)$ and $V(n)$ taken from any line may be recycled as $\underline{V}_0$ and $\underline{V}_1$ respectively in another line to form the basis of totally new explicit equations of fresh factor lines provided that they are not repeating their own line equation. For example, a pair taken thus from Factor Line 1) may be used in the explicit equations of the remaining three lines but not in their own line 1). The latter would simply be tantamount to giving a change-of-origin to a previously existing line.

A typical Factor Line

Factor Lines.

$\underline{V}n = \underline{V}_0 + n\,(\underline{V}_1 - \underline{V}_0)$



Equation of this line, $\underline{V}n = V_0 + n \cdot$ (direction vector)

Note. The direction vector can be the sum or the difference of the primary pair $\underline{V}_0$ and $\underline{V}_1$ in each equation – in this instance shown here the (direction vector) = ($\underline{V}_1 - \underline{V}_0$)). It could be in accordance with any of the four equations show above.

Claim is :- $\underline{V}_n \times \underline{V}_{(n-1)} = \underline{N}$ is true for all 'n'

This has to be proved.

Lemma – 1

The area of a triangle is half the base times the perpendicular height.

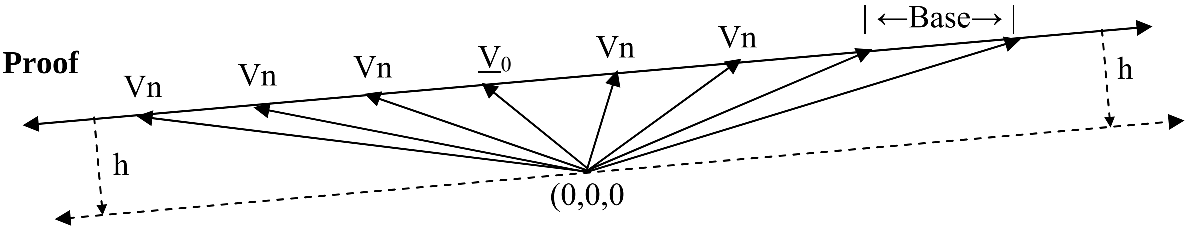
Lemma – 2

The area of a triangle is half the magnitude of the vector cross product of any two sides = half the magnitude of N in the diagram.

Lemma – 3

Vectors that have the same magnitude and direction are equal.

Proof



The construction lines together with the theory already expounded show that each triangle has the same base and the same perpendicular height => by lemma -1 these triangles are equal in area.

Lemma - 2 enables the same thing to be said in vector parlance,

$|\,\underline{V}n \times Vn\text{-}1|$ is true for all n

$|\,\underline{V}n \times Vn\text{-}1| = |\,\underline{N}\,|$ from $\underline{V}_1 \times \underline{V}_0 = \underline{N}$

- ⇨ The magnitudes of all the vector products $\underline{V}n \times Vn\text{-}1$ are equal for all 'n'.

⇨ Because all of the operands (multiplier and multiplicand) of the vector products | Vn x Vn-1| are in the same plane their respective directions are all coincident with the defining normal of the plane N and they are also equal.

By lemma – 3 , having the same magnitude and the same direction all of the vector products Vn x Vn-1 are equal to N

∴ Vn x Vn-1 = N is true for all n

The special property of the cross product that underpins the claims being made here is that the cross product of two vectors is always a third vector that has a direction that is perpendicular to the plane of the two factors i.e the plane that contains the two operands, the multiplier and multiplicand, as factors of N. This is a very, very useful property of the vector cross-product whenever it is needed in mathematics.

Comments.

1) V_0 is the natural mid-point of a factor line i.e. initially it always occurs right on the line of interception.

2) In general the mid-point of a factor line is not unique and may be varied at will.

1) All of these triangles are (totally dissimilar) 'scalene' triangles i.e. no two angles of any triangle are ever equal and no two sides are ever equal as a consequence.

2) The separate position vectors 'Vn' have coefficients that are always a co-prime set of integers.

3) The position vectors themselves are line analogues of 'n'.

4) The triangles bounded by the position vectors Vn and V(n – 1) are area analogues of 'n' in each case.

5) An inclined plane defined by N can be tiled by any one of these triangles.

These facts can be used to advantage by designers in many disciplines.
The foregoing exercise focused on the ZY intercept and initiated a factor finding scheme that emanated from that intercept. Identical working will provide similar results working from the ZX and XY intercepts and this is shown briefly in

Appendix_B and Appendix_C. All three intercepts are needed to cover the entire possibilities space of vector factors for a given vector.

If a vector for factoring has three non-zero coefficients then there will be three distinct intercepts and three sets of working will be required to find the entire set of factors for that vector.

Appendix_A - At the ZX intercept.

The equation of the plane is again,

$$\alpha \cdot x + \beta \cdot y + \gamma \cdot z = 0$$

I am looking for a point in the plane that will satisfy the equation of the plane.

I am going to guess a point that has y = 0 and then express the remaining two, each in terms of the other.

$$\alpha \cdot x + \gamma \cdot z = 0$$

For this to be true either,

1) $x = +\gamma$ and $z = -\alpha$

or

2) $x = -\gamma$ and $z = +\alpha$

These are two options here that the reader may take. I have opted for 2) and this will always be used in these notes here in future. Note, option 1) is not to be discarded completely – it can be used additionally so as to provide an extra directed number-line if that is ever required in some special case in the future.

So, $\underline{V}_0$ is written algebraically as $\begin{pmatrix} -\gamma \\ 0 \\ \alpha \end{pmatrix}$ when x is put at 0.

I do not want $\underline{V}_0$ to have any scalar take-out factor so I divide throughout by the GCD of $\{\alpha, \gamma\}$ and I shall call this GCD $'\varepsilon_y'$ (Epsilon_y <= y = 0).

$$\underline{V}_0 = \begin{pmatrix} -\gamma / \varepsilon_y \\ 0 \\ \alpha / \varepsilon_y \end{pmatrix}$$

* This is the standard form that $\underline{V}_0$ will always take in the future.

To find the cofactor $\underline{V}_1$.

$\underline{V}_1$ needs to be found next. This is the position vector of another point nearby to this $\underline{V}_0$, that is also in the plane defined by $\underline{N}$ such that $\underline{V}_1 \times \underline{V}_0 = \underline{N}$.

$$\text{Putting } V_1 = \begin{pmatrix} x \\ y \\ z \end{pmatrix} \text{ for time being}$$

$$\underline{V}_1 \text{ x } \underline{V}_0 = \underline{N} \rightarrow \begin{pmatrix} x \\ y \\ z \end{pmatrix} \times \begin{pmatrix} -\gamma/\varepsilon_y \\ 0 \\ \alpha/\varepsilon_y \end{pmatrix} = \begin{pmatrix} \alpha \\ \beta \\ \gamma \end{pmatrix}$$

$$\begin{vmatrix} i & j & k \\ x & y & z \\ -\gamma/\varepsilon_y & 0 & \alpha/\varepsilon_y \end{vmatrix} = \begin{pmatrix} \alpha \\ - \\ - \end{pmatrix}$$

Solving for the i minor of the determinant gives me all I need to know here,

$$\text{Finding } \alpha \text{ on the RHS} \rightarrow (y \times \alpha/\varepsilon_y) - (z \times 0) = \alpha$$

$$y = \varepsilon_y$$

(I need to find x and z next)

Again, by the dot product,

$$\underline{V}_1 \bullet \underline{N} \rightarrow \begin{pmatrix} x \\ y \\ z \end{pmatrix} \cdot \begin{pmatrix} \alpha \\ \beta \\ \gamma \end{pmatrix} = \alpha \cdot x + \beta \cdot y + \gamma \cdot z = 0 \text{ (the equation of the plane once more)}$$

$$1)\ x = -(\beta \cdot y + \gamma \cdot z)/\alpha \quad \text{or } 2)\ z = -(\alpha \cdot x + \beta \cdot y)/\gamma$$

Again, in this instance I also have two options as previously with finding $\underline{V}_0$

Taking option 1) and collecting terms,

$$x = -(\beta \cdot \varepsilon_y + \gamma \cdot z)/\alpha$$

$$y = \varepsilon_y$$

$$z = z$$

$$V_1 = \begin{pmatrix} -(\beta\cdot\varepsilon_y + \gamma\cdot z)/\alpha \\ \varepsilon_y \\ z \end{pmatrix} \text{ (z decides x)}$$

Altogether then, at the ZX intercept, $\underline{V}_0 = \begin{pmatrix} -\gamma/\varepsilon_y \\ 0 \\ \alpha/\varepsilon_y \end{pmatrix}$ $\underline{V}_1 = \begin{pmatrix} -(\beta\cdot\varepsilon_y + \gamma\cdot z)/\alpha \\ \varepsilon_y \\ z \end{pmatrix}$

This is the preferred form of these two 'primary' factors (not prime factors) that evolve from considering the ZX intercept only. I shall always use these in the future unless otherwise stated for all workings.

Proof that this pair V_1 x V_0 = N.

The proof is simply to multiply out $\underline{V}_1$ and $\underline{V}_0$ algebraically in the vector or cross product method of vector multiplication.

The order of multiplication is always $\underline{V}_1 \times \underline{V}_0$

$$\begin{pmatrix} -(\beta\cdot\varepsilon_y + \gamma\cdot z)/\alpha \\ \varepsilon_y \\ z \end{pmatrix} \times \begin{pmatrix} -\gamma/\varepsilon_y \\ 0 \\ \alpha/\varepsilon_y \end{pmatrix} = \begin{pmatrix} \alpha \\ \beta \\ \gamma \end{pmatrix}$$

using the conventional determinant method in the computing of a cross-product.

$$\begin{vmatrix} i & j & k \\ -(\beta\cdot\varepsilon_y + \gamma\cdot z)/\alpha & \varepsilon_y & z \\ -\gamma/\varepsilon_y & 0 & \alpha/\varepsilon_y \end{vmatrix} 1$$

$$(+)\, i \begin{vmatrix} \varepsilon_y & z \\ 0 & \alpha/\varepsilon_y \end{vmatrix} = \alpha$$

$$(-)\, j \begin{vmatrix} -(\beta\cdot\varepsilon_y + \gamma\cdot z)/\alpha & z \\ -\gamma/\varepsilon_y & \alpha/\varepsilon_y \end{vmatrix} = -\beta$$ (j minor of the determinant

$$-(\beta\cdot\varepsilon_y+\gamma.z)\big/\alpha \times \alpha\big/\varepsilon_y - ({}^{-}\gamma\cdot z\big/\varepsilon_y) = -\beta - (\gamma\cdot z)\big/\varepsilon_y + (\gamma\cdot z)\big/\varepsilon_y = -\beta$$

$-\beta$ becomes $+\beta$ with the usual change-of-sign).

$$(+)\ k\begin{vmatrix} -(\beta\cdot\varepsilon_y+\gamma\cdot z)\big/\alpha & \varepsilon_y \\ -\gamma\big/\varepsilon_y & 0 \end{vmatrix} = \gamma$$

This demonstrates that $\begin{pmatrix} -\gamma\big/\varepsilon_y \\ 0 \\ \alpha\big/\varepsilon_y \end{pmatrix}$ and $\begin{pmatrix} -(\beta\cdot\varepsilon_y+\gamma\cdot z)\big/\alpha \\ \varepsilon_y \\ z \end{pmatrix}$ are indeed de facto primary algebraic factors of N.

Appendix_B - At the XY intercept.

Finding $\underline{V}_0$ (at the XY intercept).

Once more I need to find a point in the inclined plane -
the equation of the plane to hand is,

$$(\alpha\cdot x+\beta\cdot y+\gamma\cdot z)=0$$

I will guess that there is a point in the plane that has the z coefficient = 0 (certain to be the case? when N has three non-zero coefficients).

So, $(\alpha\cdot x+\beta\cdot y+\gamma\cdot 0) \quad =0$

$$(\alpha\cdot x+\beta\cdot y)=0$$

For this to be true either,

1) y $=+\alpha$ and x = $-\beta$

or

2) y = $-\alpha$ and x = $+\beta$

These are two options here that the reader may take. I have opted for 2) and this will always be used in these notes here in future. Note, option 1) is not to be discarded completely – it can be used additionally so as to provide an extra directed number-line if that is ever required in some special case in the future.

So, $\underline{V}_0$ is written algebraically as $\begin{pmatrix} B \\ -\alpha \\ 0 \end{pmatrix}$ when z is put at 0.

I do not want $\underline{V}_0$ to have any scalar take-out factor so I divide throughout by the GCD of $\{\alpha, \beta\}$ and I shall call this GCD $'\varepsilon_z'$ (Epsilon_z <= z = 0).

$$\underline{V}_0 = \begin{pmatrix} \beta/\varepsilon_z \\ -\alpha/\varepsilon_z \\ 0 \end{pmatrix}$$

* This is the standard form that $\underline{V}_0$ will always take in the future.

To find the cofactor $\underline{V}_1$.

$\underline{V}_1$ needs to be found next. This is the position vector of another point nearby to $\underline{V}_0$, also in the plane defined by $\underline{N}$ such that $\underline{V}_1 \times \underline{V}_0 = \underline{N}$.

Putting $V_1 = \begin{pmatrix} x \\ y \\ z \end{pmatrix}$ for time being

$$\underline{V}_1 \text{ x } \underline{V}_0 = \underline{N} \rightarrow \begin{pmatrix} x \\ y \\ z \end{pmatrix} \times \begin{pmatrix} \beta/\varepsilon_z \\ -\alpha/\varepsilon_z \\ 0 \end{pmatrix} = \begin{pmatrix} \alpha \\ \beta \\ \gamma \end{pmatrix}$$

$$\begin{vmatrix} i & j & k \\ x & y & z \\ \beta/\varepsilon_z & -\alpha/\varepsilon_z & 0 \end{vmatrix} = \begin{pmatrix} \alpha \\ - \\ - \end{pmatrix}$$

Solving for the i minor of the determinant gives me all I need to know here,

$$\text{Finding } \alpha \text{ on the RHS} \rightarrow (y \cdot 0) - (-\alpha/\varepsilon_z \cdot z) = \alpha$$

$$z = \varepsilon_z$$

(I need to find y and z next)

Again, by the dot product,

$$\underline{V}_1 \bullet \underline{N} \rightarrow \begin{pmatrix} x \\ y \\ z \end{pmatrix} \cdot \begin{pmatrix} \alpha \\ \beta \\ \gamma \end{pmatrix} = \alpha \cdot x + \beta \cdot y + \gamma \cdot z = 0 \text{ (the equation of the plane once more)}$$

$$1)\ y = {-(\gamma \cdot z + \alpha \cdot x)}/{\beta} \text{ or } 2)\ x = {-(\gamma \cdot z + \beta \cdot y)}/{\alpha}$$

Again, in this instance I also have two options as previously with finding $\underline{V}_0$

Taking option 1) and collecting terms,

$$z = \varepsilon_z$$
$$x = x$$
$$y = {-(\gamma \cdot \varepsilon_z + \alpha \cdot x)}/{\beta}$$

$$\underline{V}_1 = \begin{pmatrix} x \\ -(\gamma \cdot \varepsilon_z + \alpha \cdot x)/\beta \\ \varepsilon_z \end{pmatrix} \text{ (x decides y)}$$

$$\text{Altogether then, } \underline{V}_0 = \begin{pmatrix} \beta/\varepsilon_z \\ -\alpha/\varepsilon_z \\ 0 \end{pmatrix} \quad \underline{V}_1 = \begin{pmatrix} x \\ -(\gamma \cdot \varepsilon_z + \alpha \cdot x)/\beta \\ \varepsilon_z \end{pmatrix}$$

This is the preferred form of these two 'primary' factors (not prime factors) that evolve from considering the XY intercept only. I shall always use these in the future unless otherwise stated for all workings.

Proof that this pair V_1 x V_0 = N.

The proof is simply to multiply out $\underline{V}_1$ and $\underline{V}_0$ algebraically in the vector or cross product method of vector multiplication.

The order of multiplication is always $\underline{V}_1 \times \underline{V}_0$

$$\begin{pmatrix} x \\ -(\gamma \cdot \varepsilon_z + \alpha \cdot x)/\beta \\ \varepsilon_z \end{pmatrix} \times \begin{pmatrix} \beta/\varepsilon_z \\ -\alpha/\varepsilon_z \\ 0 \end{pmatrix} = \begin{pmatrix} \alpha \\ \beta \\ \gamma \end{pmatrix}$$

using the conventional determinant method in the computing of a cross-product.

$$\begin{vmatrix} i & j & k \\ x & -(\gamma\cdot\varepsilon_z+\alpha\cdot x)/\beta & \varepsilon_z \\ \beta/\varepsilon_z & -\alpha/\varepsilon_z & 0 \end{vmatrix}$$

$$(+)\,\mathrm{i}\begin{vmatrix} -(\gamma\cdot\varepsilon_z+\alpha\cdot x)/\beta & \varepsilon_z \\ -\alpha/\varepsilon_z & 0 \end{vmatrix} = \alpha$$

$(-)\,\mathrm{j}\begin{vmatrix} x & \varepsilon_z \\ \beta/\varepsilon_z & 0 \end{vmatrix}$ = - β (the j minor of the determinant becomes $+\beta$ with change - of-sign)

$$(+)\mathrm{k}\begin{vmatrix} x & -(\gamma\cdot\varepsilon_z+\alpha\cdot x)/\beta \\ \beta/\varepsilon_z & -\alpha/\varepsilon_z \end{vmatrix} = \left(-\alpha\cdot x/\varepsilon_z\right) - \left(-(\gamma\cdot\varepsilon_z+\ \alpha\cdot x)/\beta \times \beta/\varepsilon_z\right) = \gamma$$

This demonstrates that $\begin{pmatrix} \beta/\varepsilon_z \\ -\alpha/\varepsilon_z \\ 0 \end{pmatrix}$ and $\begin{pmatrix} x \\ -(\gamma\cdot\varepsilon_z+\alpha\cdot x)/\beta \\ \varepsilon_z \end{pmatrix}$ are indeed de facto primary algebraic factors of N.

Appendix - C. A Worked Example.

Let us say that on this occasion the vector to be factored is, N = $8\hat{i}+6\hat{j}-15\hat{k}$. This 'N' is used to define a plane that passes through the origin (0, 0, 0) of the standard frame of reference frame (X, Y, Z).

Algebraically N = $\alpha\,\hat{i}+\beta\,\hat{j}+\gamma\,\hat{k}$.

$$\text{Then, } \underline{V}_0 = \begin{pmatrix} 0 \\ \gamma/\varepsilon_x \\ -\beta/\varepsilon_x \end{pmatrix} \text{ and } \underline{V}_1 = \begin{pmatrix} \varepsilon_x \\ y \\ -(\alpha\,\varepsilon_x + \beta\,y)/\gamma \end{pmatrix}$$

ε_x is the GCD of β and γ

$\underline{V}_0$ and $\underline{V}n$ are the algebraic primary factors of $\underline{N}$.

The factor line that will be used (one of the four available lines) has the vector equation,

$$\underline{V}n = \underline{V}_0 + n\,(\underline{V}_1 - \underline{V}_0)$$

The normal vector being chosen for this demonstration is,

$$\boxed{\underline{N} = \begin{pmatrix} 8 \\ 6 \\ -15 \end{pmatrix}}$$

i.e. $\alpha = 8, \beta = 6, \gamma = -15$ and $\varepsilon_x = 3$

The user factorises this $\underline{N}$ to find the primary pair $\underline{V}_0$ and $\underline{V}_1$ at the ZY axis.

$$\underline{V}_0 = \begin{pmatrix} 0 \\ \gamma/\varepsilon x \\ -\beta/\varepsilon_x \end{pmatrix} = \begin{pmatrix} 0 \\ -15/3 \\ -6/3 \end{pmatrix} = \begin{pmatrix} 0 \\ -5 \\ -2 \end{pmatrix}$$

And

$$\underline{V}_1 = \begin{pmatrix} \varepsilon_x \\ y \\ -(\alpha.\varepsilon_x + \beta.y)/\gamma \end{pmatrix} = \begin{pmatrix} 3 \\ y \\ -(8\times 3 + 6\times y)/-15 \end{pmatrix} *$$

* y decides z here - any integer value of y satisfies z

when $y = 1$ $z = {}^{-30}\!/\!_{-15} = 2$

So,

$$\underline{V}_1 = \begin{pmatrix} 3 \\ 1 \\ 2 \end{pmatrix}$$

- In the expression above, $z = -(8 \times 3 + 6 \times y) / (-15)$, y decides z and they are both integers. In this highly contrived demonstration example it is very easy using just mental arithmetic to see what integer value of y satisfies the equation but when the operands are large this becomes a very, very difficult task. The computer program to hand uses a vector factoring program that has a specially designed utility procedure that is dedicated to finding 'y' in all cases no matter how difficult so there is no problem to the human user.

Note: It is good practice to always check that,
$\underline{V}_1$ x $\underline{V}_0$ = $\underline{N}$, after finding those two.

$$\underline{V}_0 = \begin{pmatrix} 0 \\ -5 \\ -2 \end{pmatrix} \text{ and } \underline{V}_1 = \begin{pmatrix} 3 \\ 1 \\ 2 \end{pmatrix}$$

$$\begin{pmatrix} 3 \\ 1 \\ 2 \end{pmatrix} \text{x} \begin{pmatrix} 0 \\ -5 \\ -2 \end{pmatrix} = \begin{pmatrix} 8 \\ 6 \\ -15 \end{pmatrix}$$

They are true factors!

Reminder, the chosen line is,

$$\underline{V}n = \underline{V}_0 + n\,(\underline{V}_1 - \underline{V}_0)$$

$$(\underline{V}_1 - \underline{V}_0) \text{ is } \begin{pmatrix} 3 \\ 1 \\ 2 \end{pmatrix} - \begin{pmatrix} 0 \\ -5 \\ -2 \end{pmatrix} = \begin{pmatrix} 3 \\ 6 \\ 4 \end{pmatrix}$$

Explicitly then,

The chosen line is,

$$\underline{V}n = \begin{pmatrix} 0 \\ -5 \\ -2 \end{pmatrix} + n \begin{pmatrix} 3 \\ 6 \\ 4 \end{pmatrix}$$

The position vector $\underline{V}n$ of any number 'n' can be found by substituting in the appropriate value of 'n' in the RHS.

Let us say that n = 489. => n-1 = 488

Substituting 'n' into the equation of the number line,

$$\underline{V}n = \begin{pmatrix} 0 \\ -5 \\ -2 \end{pmatrix} + n \begin{pmatrix} 3 \\ 6 \\ 4 \end{pmatrix}$$

$$\Rightarrow \underline{P}\,489 = \begin{pmatrix} 0 \\ -5 \\ -2 \end{pmatrix} + 489 \begin{pmatrix} 3 \\ 6 \\ 4 \end{pmatrix}$$

(The position vector is given the variable name P now)

$$\Rightarrow \underline{P}\,489 = \begin{pmatrix} 1467 \\ 2929 \\ 1954 \end{pmatrix}$$

$$\Rightarrow P(n-1) = \underline{P}\,488 = \begin{pmatrix} 0 \\ -5 \\ -2 \end{pmatrix} + 488 \begin{pmatrix} 3 \\ 6 \\ 4 \end{pmatrix}$$

$$\Rightarrow P488 = \begin{pmatrix} 1464 \\ 2923 \\ 1950 \end{pmatrix}$$

Checking that $\underline{P}n$ x $\underline{P}(n-1) = \underline{N}$,

$$\begin{pmatrix} 1467 \\ 2929 \\ 1954 \end{pmatrix} \text{x} \begin{pmatrix} 1464 \\ 2923 \\ 1950 \end{pmatrix} = \begin{pmatrix} 8 \\ 6 \\ -15 \end{pmatrix} \text{ - correct.}$$

* $\underline{P}n$ and P(n-1) are an ordered pair of bona fide factors of $\underline{N}$

(This example is related to vector cryptography). When a seeding pair is found then the explicit equations of multiple factor lines may be found according to the equations described.

Appendix_D. Discussion.

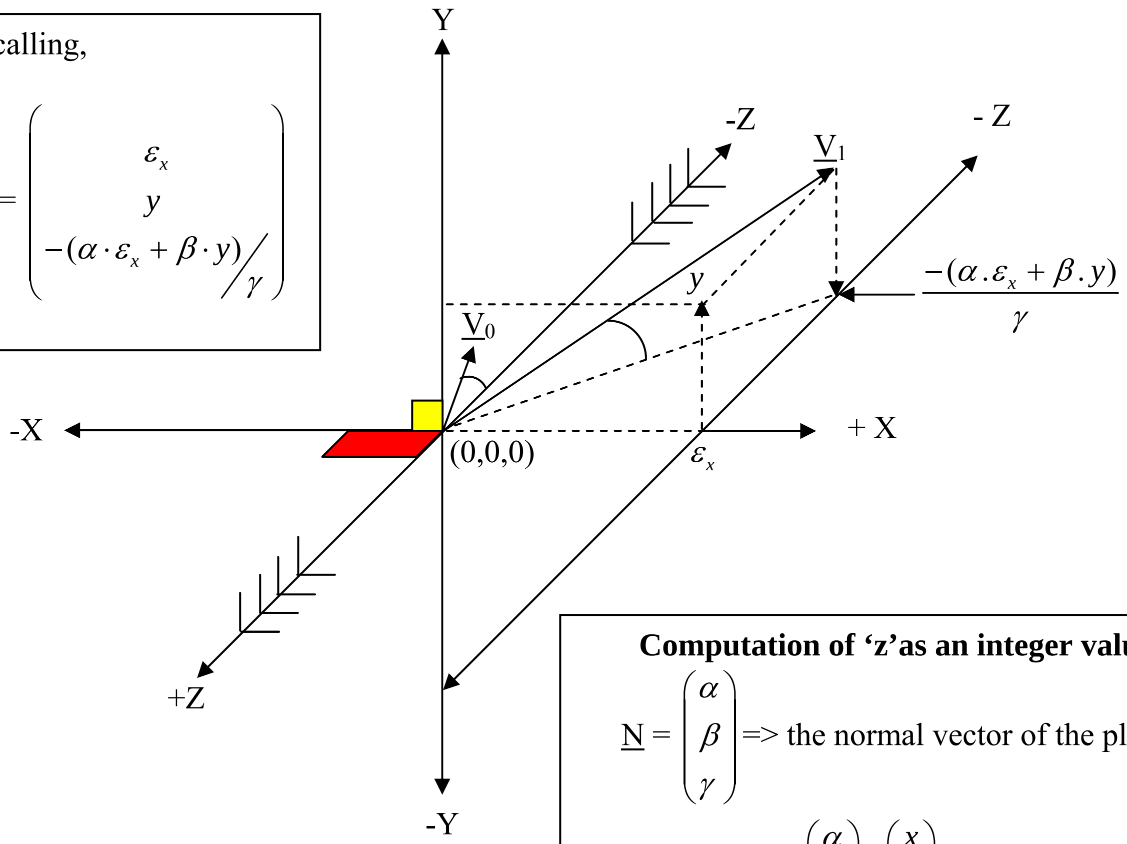
The application to cryptography ushers in displacement in three-dimensional space as a future basis for main stream encryption algorithms that use vector methodology. This is not the only application of vector factoring and it is very likely that there will be other currently unforeseen uses for it in the sciences and technologies that use vectors a lot such as vector mechanics, in the future. It is thought to have potential as a tool for design engineers and physicists.

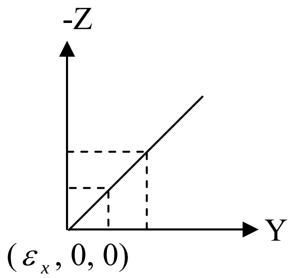
Appendix_E. Useful Laminates follow.

Demonstrating V_0 and V_1 Graphically.

Recalling,

$$\underline{V}_1 = \begin{pmatrix} \varepsilon_x \\ y \\ -(\alpha \cdot \varepsilon_x + \beta \cdot y)/\gamma \end{pmatrix}$$





Integer y for integer z

Computation of 'z'as an integer value

$$\underline{N} = \begin{pmatrix} \alpha \\ \beta \\ \gamma \end{pmatrix} \Rightarrow \text{the normal vector of the plane}$$

$$\begin{pmatrix} \alpha \\ \beta \\ \gamma \end{pmatrix} \cdot \begin{pmatrix} x \\ y \\ z \end{pmatrix} = 0$$

$\alpha.\varepsilon_x$ is known , and let 'y 'vary in $\beta \cdot$y then,

$$z = -(\alpha.x + \beta.y)/\gamma$$

(i.e. y decides z here)

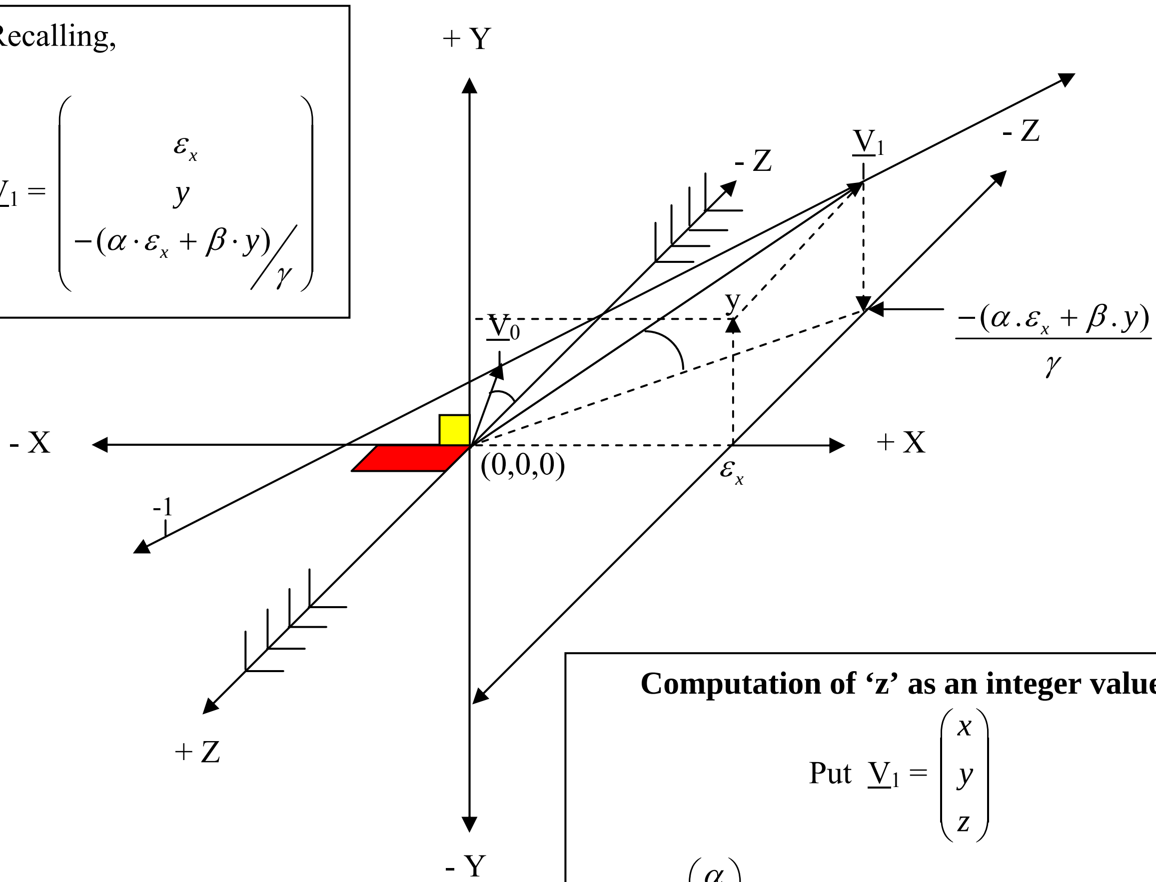
'y' is progressively increased (or decreased) in steps of 1 in the computer program until the outcome of the equation is an integer value on the Z axis. Any value of 'y' that does this is in order – not necessarily the first. The only thing to watch is that the ensuing computations at a later stage of the encryption/decryption process do not become too large for the computer to store as integers.

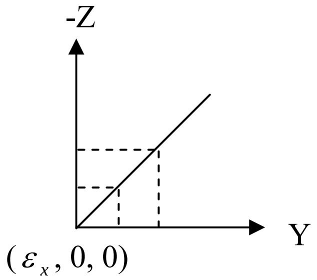
Demonstrating V_0 and V_1 Graphically.

V_0 and V_1 & a number-line that they might define

Recalling,

$$\underline{V}_1 = \begin{pmatrix} \varepsilon_x \\ y \\ -(\alpha \cdot \varepsilon_x + \beta \cdot y)/\gamma \end{pmatrix}$$





Integer y for integer z

Computation of 'z' as an integer value

Put $\underline{V}_1 = \begin{pmatrix} x \\ y \\ z \end{pmatrix}$

$\underline{N} = \begin{pmatrix} \alpha \\ \beta \\ \gamma \end{pmatrix}$ => the normal vector of the plane

$$\begin{pmatrix} \alpha \\ \beta \\ \gamma \end{pmatrix} \cdot \begin{pmatrix} x \\ y \\ z \end{pmatrix} = 0$$

$\alpha.\varepsilon_x$ is known , and let 'y 'vary in $\beta \cdot$ y then,

$$z = -(\alpha.x + \beta.y)/\gamma$$

(i.e. y decides z here)

'y' is progressively increased (or decreased) in steps of 1 in the computer program until the outcome of the equation is an integer value on the Z axis. Any value of y is in order – not necessarily the first. The only thing to watch is that the ensuing computations at a later stage of the encryption/decryption process do not become too large for the computer to store as integers.

Vector Cryptography – Discussion Model

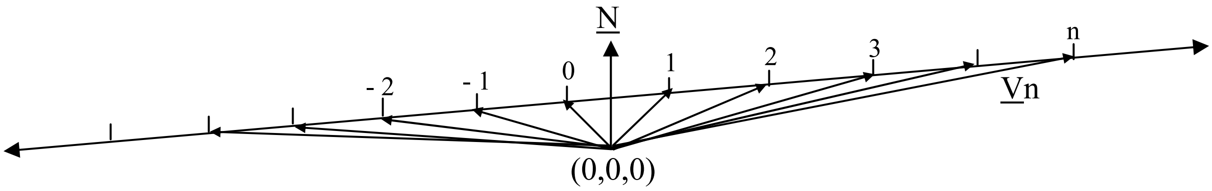
$Let\ \underline{N} = (\alpha.\hat{i} + \beta.\hat{j} + \gamma.\hat{k}) \equiv \begin{pmatrix} \alpha \\ \beta \\ \gamma \end{pmatrix}$ Let, $\varepsilon_x = GCD\{\beta,\gamma\}$, $\varepsilon_y = GCD\{\alpha,\gamma\}$, $\varepsilon_z = GCD\{\alpha,\beta\}$

$Let\ \alpha : +ve\ \text{int}, 10 <= \alpha <= 999$

$Let\ \beta : +ve\ \text{int}, 10 <= \beta <= 999$

$Let\ \gamma : +ve\ \text{int}, 10 <= \gamma <= 999$

Encryption Line.



$\underline{V}n = \underline{V}_0 + n(\underline{V}_1 - \underline{V}_0)$

N kick-starts the encryption transformation, it is used only once before being replaced for the next item.

$\underline{V}_0$ is the position vector of the number '0' on the directed number-line shown.

$\underline{V}_1$ is the position vector of the number '1' on the directed number-line shown.

The number line here is defined by the fact that it passes through these two known points.

$\underline{V}_0$ and $\underline{V}_1$ are a primary or 'seeding' pair of factors of N.

$\underline{V}_1$ (pronounced VeeOne) is the cofactor of $\underline{V}_0$ (VeeZero) such that $\underline{V}_1 \times \underline{V}_0 = \underline{N}$ when multiplied out in that order in the vector or 'cross' product method of vector multiplication.

Subsequently, any pair $\underline{V}n$ and V_{n-1} on any factor line (related lines derived later) are vector factors of N when taken in this same order.

Although discussed as pairs of factors here, $\underline{V}_0$ and $\underline{V}_1$ are each distinct 'standalone' factors of N in their own right.

There are three options for $\underline{V}_0$ and six options for $\underline{V}_1$ in defining equations of derived factor lines.

The integer representation (codepoint) of each plaintext currently being encrypted is assigned to this line; the position vector of the number on the line then becomes a reversible analogue of the number in question.

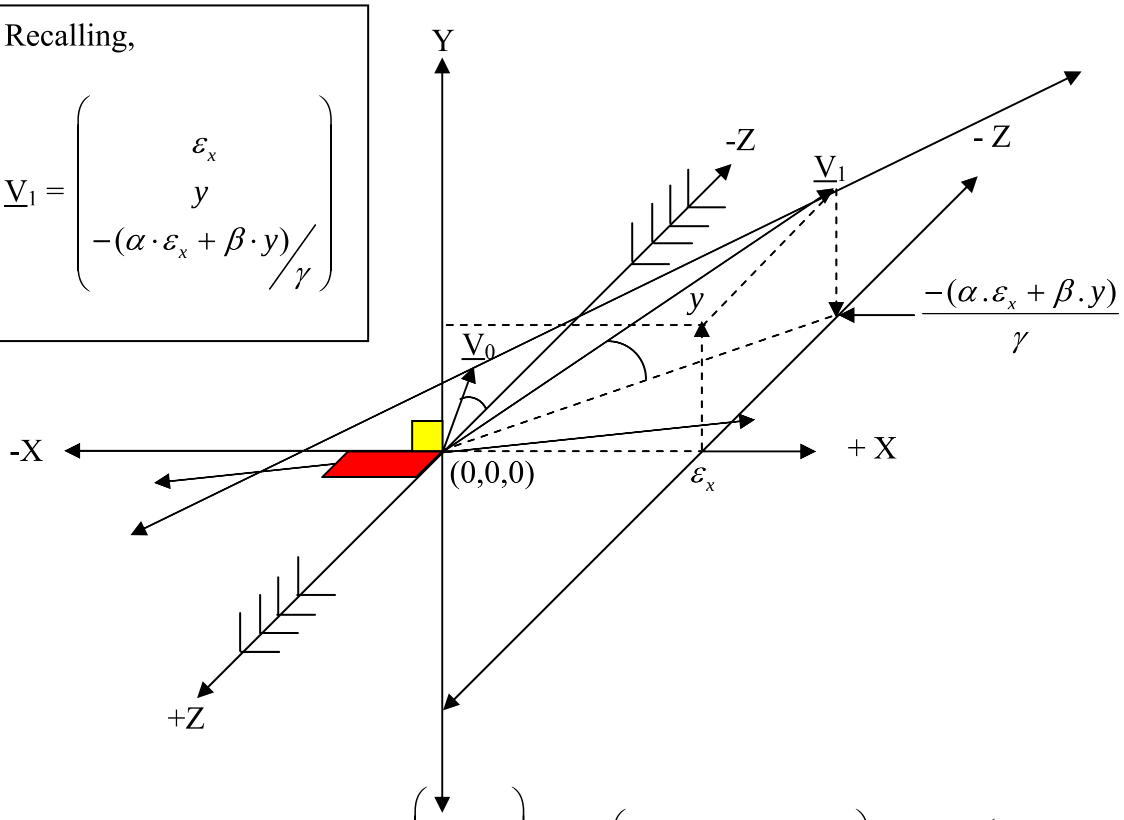
The line through N, i.e. the line that has the same direction as N and the inclined number line now comprise a pair of orthogonal skew lines. The upshot of this fact is that the vector or cross product of pairs of position vectors of numbers on the inclined number line (the exact details of which need not be known) can conveniently be used to rotate the important information, i.e.the 'n' of Pn, onto the pairing skew line through N where it can be easily read. The vector or crossproduct has a characteristic transforming effect in that the resulting product of the multiplier and the multiplicand of two vectors is always a third vector that is at 90 degrees to the plane of the first two when they are all taken in the correct order of a right-handed triad. This latter caveat is essential in order to qualify uniquely.

Vector cryptography makes heavy use of this very useful fact.

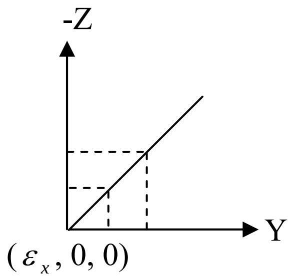
Demonstrating V_0 and V_1 Graphically.
And a Number Line They Might Define.

Recalling,

$$\underline{V}_1 = \begin{pmatrix} \varepsilon_x \\ y \\ -(\alpha \cdot \varepsilon_x + \beta \cdot y)/\gamma \end{pmatrix}$$



$$\underline{V}_0 = \begin{pmatrix} 0 \\ \gamma/\varepsilon_x \\ -\beta/\varepsilon_x \end{pmatrix} \quad \underline{V}_1 = \begin{pmatrix} \varepsilon_x \\ y \\ -(\alpha.\varepsilon_x + \beta.y)/\gamma \end{pmatrix} \text{ or } \underline{V}_1 = \begin{pmatrix} \varepsilon_x \\ -(\alpha.\varepsilon_x + \gamma.z)/\beta \\ z \end{pmatrix}$$



Integer y for integer z

$$\underline{V}_0 = \begin{pmatrix} -\gamma/\varepsilon_y \\ 0 \\ \alpha/\varepsilon_y \end{pmatrix} \quad \underline{V}_1 = \begin{pmatrix} -(\beta.\varepsilon_y + \gamma.z)/\alpha \\ \varepsilon_y \\ z \end{pmatrix} \text{ or } \underline{V}_1 = \begin{pmatrix} x \\ \varepsilon_y \\ -(\beta.\varepsilon_y + \alpha.x)/\gamma \end{pmatrix}$$

Equations of lines that may be used as factor lines.

$\underline{V}n = \underline{V}_0 + n(\underline{V}_1 - \underline{V}_0)$

$\underline{V}n = \underline{V}_0 + n(\underline{V}_1 + \underline{V}_0)$

$\underline{V}n = \underline{V}_0 + n(\underline{V}_1)$

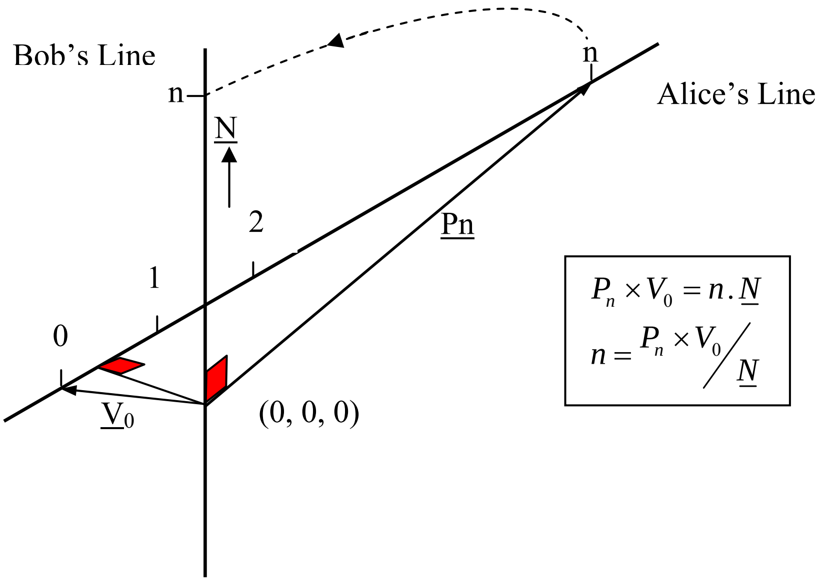
$\underline{V}n = \underline{V}_1 + n(-V_0)$

$$\underline{V}_0 = \begin{pmatrix} \beta/\varepsilon_z \\ -\alpha/\varepsilon_z \\ 0 \end{pmatrix} \quad \underline{V}_1 = \begin{pmatrix} x \\ -(\gamma.\varepsilon_z + \alpha.x)/\beta \\ \varepsilon_z \end{pmatrix} \text{ or } \underline{V}_1 = \begin{pmatrix} -(\gamma.\varepsilon_z + \beta.y)/\alpha \\ y \\ \varepsilon_z \end{pmatrix}$$

The Talking Transformer.

This is a parody on the way in which Alice and Bob communicate using vector cryptography. That is by means of a geometric tool used for 'distance communication' in a vector cipher. Alice's parameters are number inputs to the 'tool' that enable Bob to understand what she is saying cryptographically. The scheme makes use of vector factoring and orthogonal skew lines.

The model is a pair of orthogonal skew lines that are related to each other in the amount of skew by means of the factoring of vectors methodology already described.



$$P_n \times V_0 = n.\underline{N}$$
$$n = \frac{P_n \times V_0}{\underline{N}}$$

Any number that Alice puts or her line, Bob can transfer and 'read' it by rotating it on to his own line. The mathematical transfer operator that enables this to happen is the simple cross-product.

Alice creates a line to her liking and factorises it etc. and finds $\underline{P}n$ for the current character that she is enciphering. She gives this a hefty change-of-origin and sends it to Bob (Please see the Discussion Model for a view of her cipher-text string). Bob removes the change-of-origin to uncover the real origin and then,

$$\underline{P}n \text{ x } \underline{V}_0 = n.\underline{N}$$

(in words just to clarify this, $\underline{P}$n cross VeeZero = n times $\underline{N}$)

Dividing corresponding coefficients left and right of the equals sign gives 'n' for decoding back to its ASCII evaluation by Bob.

Appendix_E. Some Vector Arithmetic Used here.

A vector written horizontally in component form may conveniently be also written in vertical column form:

$$\underline{V} = \hat{i} + \hat{j} + \hat{k} \text{ is the same as } \underline{V} = \begin{pmatrix} i \\ j \\ k \end{pmatrix}$$

In these examples $\underline{V}_1 = \begin{pmatrix} a \\ b \\ c \end{pmatrix}$ and $\underline{V}_2 = \begin{pmatrix} d \\ e \\ f \end{pmatrix}$

Vector Multiplication or the Vector Product of two vectors is so-called because the product is always a vector (this form of multiplication is also called the cross product). Importantly this operation is non-commutative. That means the operands must be written in a particular order to get the correct result.

It means that ($\underline{V}_1$ x $\underline{V}_2$) /= ($\underline{V}_2$ x $\underline{V}_1$)

When the multiplicand and the multiplier are changed around then the product becomes the (–ve) of what it was prior to the change so that,

$$(\underline{V}_2 \text{ x } \underline{V}_1) = -(\underline{V}_1 \text{ x } \underline{V}_2)$$

$$\begin{pmatrix} a \\ b \\ c \end{pmatrix} \text{x} \begin{pmatrix} d \\ e \\ f \end{pmatrix} = \begin{pmatrix} (bf - ce) \\ -(af - cd) \\ (ae - bd) \end{pmatrix}$$

A useful rule for finding the Cross Product directly by inspection,

$$\begin{pmatrix} a \\ b \\ c \end{pmatrix} \text{x} \begin{pmatrix} d \\ e \\ f \end{pmatrix} = \begin{pmatrix} - \\ - \\ - \end{pmatrix}$$

> To find a coefficient (-) on the RHS cover the corresponding row on the LHS (use your ballpoint pen to do this) and mentally solve the determinant formed by the remaining two rows. Don't forget the change of sign with the middle one.

Example,

$$\begin{pmatrix} 4 \\ 2 \\ -1 \end{pmatrix} \text{ x } \begin{pmatrix} 3 \\ -2 \\ 1 \end{pmatrix} = \begin{pmatrix} 0 \\ -7 \\ -14 \end{pmatrix}$$

Multiplication of a vector by a scalar.

Simply multiply each coefficient by the scalar,

$$\begin{pmatrix} a \\ b \\ c \end{pmatrix} \text{ x 5 (say)} = \begin{pmatrix} (5a) \\ (5b) \\ (5c) \end{pmatrix}$$

Scalar multiplication or Dot Product of two vectors.

Simply multiply corresponding coefficients and add the three products.

$$\begin{pmatrix} a \\ b \\ c \end{pmatrix} \bullet \begin{pmatrix} d \\ e \\ f \end{pmatrix} = \text{(ad) + (be) + (cf)} \quad \text{Example, } \begin{pmatrix} 3 \\ 5 \\ 8 \end{pmatrix} \bullet \begin{pmatrix} -7 \\ 2 \\ 1 \end{pmatrix} = -21 + 10 + 8 = -3$$

(note: the dot product is always a scalar)

Addition of two Vectors

Simply add corresponding coefficients,

$$\begin{pmatrix} a \\ b \\ c \end{pmatrix} + \begin{pmatrix} d \\ e \\ f \end{pmatrix} = \begin{pmatrix} (a+d) \\ (b+e) \\ (c+f) \end{pmatrix} \quad \text{Example, } \begin{pmatrix} 6 \\ 3 \\ 1 \end{pmatrix} + \begin{pmatrix} 2 \\ 3 \\ -2 \end{pmatrix} = \begin{pmatrix} 8 \\ 6 \\ -1 \end{pmatrix}$$

Subtraction of two vectors

$$\begin{pmatrix} a \\ b \\ c \end{pmatrix} - \begin{pmatrix} d \\ e \\ f \end{pmatrix} = \begin{pmatrix} (a-d) \\ (b-e) \\ (c-f) \end{pmatrix} \quad \text{Example, } \begin{pmatrix} -4 \\ 6 \\ 9 \end{pmatrix} - \begin{pmatrix} -2 \\ 4 \\ 5 \end{pmatrix} = \begin{pmatrix} -2 \\ 2 \\ 4 \end{pmatrix}$$

Vector Division is Undefined

There is no general algorithm for it in everyday mathematics.

Austin O'Byrne.

October 2014.